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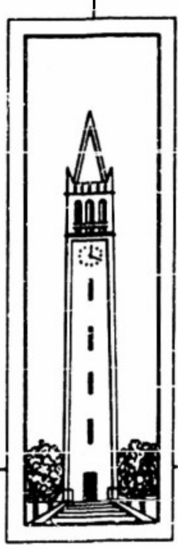
BY

R. C. MacCAMY

OCTOBER 1953



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1. Introduction

Attempts thusfar to treat the motion of ships in ocean waves have been almost exclusively limited to semi-empirical methods based on the Froude-Kriloff hypothesis. That is, the effect of the ship on the indicent wave is assumed to be contained in two terms involving first and second derivatives multiplied by constants called the damping factor and virtual mass respectively. These constants are then determined experimentally by oscillation of the object in still water. The forcing function of the resulting differential equation of motion is then taken as the pressure due to the undisturbed incident wave.

The procedure just outlined has been presented in detail by Fuchs[(1)*] for the case of a rectangular block and simple harmonic motion, and the results generalized by Fuchs and MacCamy[(2)] to the case of non-periodic motions. It was felt by the author, therefore, that such considerations had been carried sufficiently far to warrant a more critical analysis of the problem of ship motion. To this end a study of the hydrodynamic problems involved was begun, with the ultimate goal of obtaining solutions which are complete within themselves and do not depend on experimental data. In order to deal with the simplest cases, mathematically, the investigations have been limited to the two dimensional problem of an infinitely long circular cylinder and the three dimensional problem of a sphere.

The present report represents the first results of the above study and deals exclusively with the cylinder. A portion of this problem has been considered by Ursell[(3)], namely, the forced oscillation of the cylinder. Ursell's results yield the veritcal force on the cylinder in a form exactly like that described above except that the virtual mass and damping factors are now functions of frequency. From this solution we obtain an approximation of the heaving motion in incident waves by neglecting the diffraction by the moving cylinder. This approximate solution is found to be valid for waves which are long compared to the diameter of the cylinder.

The method used here is essentially that recommended by Weinblum and St. Denis[(4)]. The idea of introducing virtual mass and damping factors, as determined from the forced oscillation of the object was investigated by Havelock[(5)]. He succeeded in obtaining an approximation to the damping factor by assuming the object might be replaced, in forced oscillation, by an equivalent distribution of sources.

Another approximate solution, obtained by an entirely different method, is found to yield good agreement with observed values provided the waves are

* For figures in parenthesis see References at end of report.

short compared to the diameter. This method involves replacing the usual free surface condition for the velocity potential, ϕ, by the simpler condition, $\phi = 0$. The condition, $\phi = 0$, has been previously used by J.L. Taylor(6) and others; again, however, only for the case of forced oscillation. The present report appears to be the first effort to extend the method to treat freely floating objects. A discussion of this simplified boundary condition is given in the appendix, and it is shown that it is indeed the proper approximation to use in considering large frequencies, i.e., short waves.

Using these two approximate solutions it is found to be possible to predict the heaving motion of the cylinder with good accuracy over almost the entire range of frequencies considered.

2. Forced Oscillation of a Circular Cylinder

We suppose a fluid, infinitely deep, occupying the region $-\infty \leq x \leq +\infty, 0 < y < \infty$ so that $y = 0$ is a free surface. The problem treated by Ursell is the following. An infinitely long circular cylinder of radius a, is placed with its center at the origin, $(0, 0)$, and made to oscillate vertically with simple harmonic motion of small amplitude. We will not review Ursell's procedure in detail, but will content ourselves with giving the results, which are needed for the subsequent developments. Briefly, Ursell obtains a solution by assuming the velocity potential may be represented as a superposition of the potential due to a point source at the origin, and an infinite series of non-orthogonal harmonic functions, satisfying the free surface condition. A method for computing the coefficients in the expansion is given and a proof given for the convergence of the series.

For our purpose the important result of Ursell's work is the vertical force per unit width on the cylinder. It is found that if ζ represents the heaving motion, the force may be written in the form

$$F_v(t) = M'\left(\frac{\sigma^2 a}{g}\right)\frac{d^2\zeta}{dt^2} + N\left(\frac{\sigma^2 a}{g}\right)\frac{d\zeta}{dt} + 2\mu g a \zeta \tag{1}$$

where σ is the frequency of the motion, μ the density of fluid and g the acceleration of gravity. The functions M' and N are given by

$$M'\left(\frac{\sigma^2 a}{g}\right) = \frac{1}{2}\pi\mu a^2 m\left(\frac{\sigma^2 a}{g}\right);$$
$$N\left(\frac{\sigma^2 a}{g}\right) = 2\mu\sigma a^2 \frac{M_0(\sigma^2 a/g)A(\sigma^2 a/g) - N_0(\sigma^2 a/g)B(\sigma^2 a/g)}{A^2(\sigma^2 a/g) + B^2(\sigma^2 a/g)}, \tag{2}$$

with m, A, B tabulated functions. The functions M_0 and N_0 are not tabulated explicitly but may be easily determined by equating the average work done over a cycle to the average rate of propagation of energy outward. This condition leads to

$$M_0 A - N_0 B = \frac{1}{2}\pi^2$$

which together with the expression for m,

$$m = \frac{M_0 B + N_0 A}{A^2 + B^2} ,$$

permits the calculation of M_0 and N_0 and hence the second term in the force.

It is to be noted that the force as given by Equation (1) has the same form as has been assumed in previous reports. However, the quantities M' and N are no longer constants, but are functions of frequency. These functions are plotted in Figure 1, with the values obtained by oscillation in still water indicated. We see that although the experimental values do not differ too much from the theoretical values at the same frequency, they differ quite radically from theoretical values at other frequencies.

3. Heaving Motion for Long Waves

To obtain an approximate solution to the problem of finding the motion of the cylinder in an incident wave train, we proceed now along the lines used by Fuchs(1) in an earlier report. That is, we assume the heaving to be periodic with the same frequency as the incident wave, and we add to the force already obtained, the force due to the undisturbed incident wave. The resulting differential equation for ζ differs from that given by Fuchs in that the coefficients of the first and second derivatives are functions of frequency. Such an assumption means that we are neglecting the effect of diffraction of the incident wave by the moving cylinder.

For purposes of calculation we introduce polar co-ordinates (r, θ) where r is measured radially outward from the origin and θ is measured from the positive y axis (which is vertically downward). The velocity potential of the incident wave is

$$\phi = \frac{g A}{\sigma} e^{-ky} \cos (kx - \sigma t) ,$$

where A is the amplitude and $k = \frac{\sigma^2}{g}$. From Bernoulli's equation the periodic part of the pressure is

$$p = \mu g \frac{\partial \phi}{\partial t}$$

and the resulting vertical force per unit width on the cylinder is

$$F_v^{(i)}(t) = a \mu g A \int_{-\pi/2}^{+\pi/2} e^{-ka \cos\theta} \sin (ka \sin\theta - \sigma t) \cos\theta \, d\theta$$

$$= -2 a \mu g A \sin \sigma t \int_0^{\pi/2} e^{-ka\cos\theta} \cos (ka \sin\theta) \cos\theta \, d\theta .$$

Now

$$e^{-ka\cos\theta} \cos (ka \sin\theta) = \mathrm{Re}\,(e^{-ka e^{i\theta}}) = \mathrm{Re} \sum_{n=0}^{\infty} \frac{(-1)^n (ka)^n}{n!} e^{in\theta}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (ka)^n}{n!} \cos n\theta ;$$

thus, integrating term by term, we find for the force,

$$F_v^{(1)}(t) = -2a\mu g A \sin\sigma t \sum_{n=0}^{\infty} \frac{(-1)^n \ (ka)^n}{n!} \int_0^{\pi/2} \cos n\theta \cos\theta \, d\theta$$

$$= -2a\mu g A \sin\sigma t \ f(ka) ,$$

where

$$f(ka) = 1 - \frac{\pi}{4} ka + \frac{1}{6} (ka)^2 - \frac{1}{(15)(24)} (ka)^4 + \frac{1}{(35)(720)} (ka)^6 \cdots . \quad (3)$$

With the notation

$$M\left(\frac{\sigma^2 a}{g}\right) = \frac{1}{2} \pi a^2 \mu \left(1 + m\left(\frac{\sigma^2 a}{g}\right)\right) ,$$

$$2b = \frac{N(\sigma^2 a/g)}{M(\sigma^2 a/g)} , \qquad \omega^2 = \frac{2\mu g a}{M(\sigma^2 a/g)} , \qquad f_0 = -\frac{2a\mu g}{M(\sigma^2 a/g)}$$

the differential equation of motion becomes

$$\frac{d^2\zeta}{dt^2} + 2b\frac{d\zeta}{dt} + \omega^2 \zeta = f_0 A f(ka) \sin\sigma t .$$

The steady state solution is

$$\zeta = \frac{f_0 \ f(ka) \ A}{(\omega^2 - \sigma^2)^2 + 4b^2\sigma^2} \left[(\omega^2 - \sigma^2)\sin\sigma t + 2b\sigma\cos\sigma t\right] \quad (4)$$

so that the relative heaving amplitude is

$$|\zeta/A| = \frac{f_0 \ f(ka)}{\sqrt{(\omega^2 - \sigma^2)^2 + 4b^2\sigma^2}} . \quad (5)$$

This ratio is plotted in Figure 2 with comparison to the observed values obtained by Sibul(7). The agreement is seen to be quite good for the first portion of the curve where the waves are long compared to the diameter of the cylinder, but the theoretical results drop off too rapidly as the wave length shortens. The theory does, however, seem to be adequate for the region of maximum heaving.

4. Heaving Motion for Short Waves

In discussing wave motion involving a free surface, at say $y = 0$, one has to deal, to a first approximation, with the boundary condition

$$\sigma^2\phi - g\frac{\partial\phi}{\partial y} = 0 \quad \text{or} \quad y = 0 .$$

It is shown in the appendix, however, that for large frequencies, this condition reduces to the simpler one $\phi = 0$ on $y = 0$. We are led then to search for a function harmonic in the domain of the fluid, vanishing on the portion of $y = 0$ corresponding to the free surface, and having a prescribed normal derivative on the cylinder.

This simplified problem may be solved by straightforward methods of separation of variables. We prefer to give the solution in terms of a Green's function having a logarithmic singularity. The latter method is, of course, exactly equivalent to the former insofar as the formal calculations are concerned, but the modification of sources to satisfy the complete free surface condition is known[8], and it is felt that the present method might serve as a first step in a series solution to the complete problem. That is, one could get a second approximation which satisfied the free surface condition but no longer satisfied the condition at the cylinder; then correct again for the condition at the cylinder, etc. Such a technique has been used with some success by Havelock[9] to study the motion produced by completely submerged objects.

It will be convenient for this section to measure the polar angle, θ, positively clockwise from the positive x-axis. Let $P = (r, \psi)$, $Q = (\rho, \theta)$ represent two points, and R be the distance between them. Consider the function $G_1(P, Q)$ defined by

$$G_1(P, Q) = -\log r - \log \frac{1}{\sqrt{r^2 + \rho^2 - 2r\rho\cos(\theta - \psi)}} - \log \frac{1}{\sqrt{r^2 + \frac{a^4}{\rho^2} - 2r\frac{a^2}{\rho}\cos(\theta - \psi)}} \,. \tag{6}$$

$G_1(P, Q)$ is harmonic for $P \neq Q$ and has a logarithmic singularity for $P = Q$. Using the identity

$$\log\left\{\frac{1}{\sqrt{1 - 2r\cos x + r^2}}\right\} = \sum_{n=0}^{\infty} \frac{1}{n} r^n \cos nx \,, \tag{7}$$

we find

$$G_1 = \log\rho - \sum_{n=1}^{\infty} \frac{1}{\rho^n}\left\{r^n + \frac{a^{2n}}{r^n}\right\}\cos n(\theta - \psi)$$

from which it follows that

$$\frac{2G_1}{2r} = 0 \quad \text{at} \quad r = a \,.$$

If we define

$$G(P, Q) = \log\left\{\frac{\sqrt{r + \rho - 2r\rho\cos(\theta + \psi)}}{\sqrt{r + \rho - \ r\rho\cos(\theta - \psi)}}\right\} + \log\left\{\frac{\sqrt{r + a/\rho - 2a\ r/\rho\cos(\theta + \psi)}}{\sqrt{r\ \ a/\rho - 2a\ r/\rho\cos(\theta - \psi)}}\right\} \tag{8}$$

we have the desired Green's function. That is, $G(P, Q)$ satisfies

A) $G(P, Q)$ harmonic for $P \neq a$ in $-\infty \leq x \leq +\infty$ $0 < y < \infty$;

B) $G - \log R$ regular in the same region;

C) $G = 0$ on $y = 0$;

D) $\frac{\partial G}{\partial \rho} = 0$ on $\rho = a$.

We remark further that

$$G(r, \psi, \rho, \theta) = O(1/\rho) \quad \text{as} \quad \rho \to \infty \,. \tag{9}$$

Furthermore the conditions that ϕ be bounded at ∞ and be 0 on $y = 0$ demand that

$$\phi(\rho, \theta) = O(1/\rho) \qquad \text{as} \qquad \rho \to \infty . \tag{10}$$

We are now in a position to express the velocity potential ϕ satisfying the above mentioned conditions. We apply Green's theorem,

$$\int_D\int (u \nabla^2 v - v \nabla^2 u) \, dS = \int_C \left(u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n} \right) dt ,$$

where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$, D is a region in the plane, C is its boundary, and n is the exterior normal to C. For D we take the region bounded by the cylinder, the line $y = 0$, a large semi-circle of radius K and center at the origin, and a small circle of radius ϵ and center at (r, ψ). Then both G and ϕ are harmonic in D and Green's theorem applies, with the left hand side equal to zero. Using conditions (9) and (10) the integral over large semi-circle vanishes as $K \to \infty$. Using (B) the integral over the small circle tends to $2\pi \phi(r, \psi)$ as $\epsilon \to 0$. Consequently, using (C) and (D), and letting $K \to \infty$, $\epsilon \to 0$,

$$\phi(r, \psi) = \frac{1}{2\pi} \int_C \frac{\partial \phi}{\partial \rho}(\rho, \theta) \, G(r, \psi, \rho, \theta) \, d\tau_c , \tag{11}$$

where C_c is the lower half of the cylinder. Again using the identity (7) we find

$$\begin{aligned} G(r, \psi, a, \theta) &= 2 \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{a}{r}\right)^n \cos n(\theta + \psi) - 2 \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{a}{r}\right)^n \cos n(\theta - \psi) \\ &= -4 \sum_{1}^{\infty} \frac{1}{n} \left(\frac{a}{r}\right)^n \sin n\theta \sin n\psi . \end{aligned} \tag{12}$$

From Equations (11) and (12),

$$\phi(r, \psi) = -\frac{2a}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{a}{r}\right)^n \sin n\psi \int \frac{\partial \phi}{\partial \rho}(a, \theta) \sin n\theta \, d\theta . \tag{13}$$

The pressure is obtained from

$$p(r, \psi, t) = \mu \frac{\partial \phi}{\partial t} .$$

Hence, for the vertical force on the cylinder we have

$$F_v(t) = \mu a \int_0^{\pi} \left(\frac{\partial \phi}{\partial t} \right)_{n=a} \sin \psi \, d\psi = -\mu a^2 \int_0^{\pi} \left(\frac{\partial^2 \phi}{\partial \rho \, \partial t} \right)_{n=a} \sin \theta \, d\theta . \tag{14}$$

To make the connection between our results and the theory of sources in a fluid we need only note that the function $G_1(P, Q)$ as given by Equation (6),

is merely the superposition of line sources of unit strength at (ρ, θ) and the inverse point with respect to the circle, i.e., $(a^2/\rho, \theta)$, and a sink of strength one at the origin. If we modify these sources as indicated by Kennard(8) we will have a function satisfying the complete free surface condition but no longer having zero normal derivative on the cylinder.

We consider two cases. First the forced oscillation of the cylinder with a velocity $u(t)$. Then $\left(\frac{\partial^2 \phi}{\partial \rho \partial t}\right)_{\rho=a} = \frac{\partial u}{\partial t} \sin\theta$. From Equation (14),

$$F_v^{(1)}(t) = -\frac{1}{2}\pi\mu a^2 \frac{\partial u}{\partial t}. \tag{15}$$

This result is a familiar one, being the force on the lower half of a cylinder moving vertically with velocity u, in an infinite fluid. The second case is to correspond to the diffracted wave produced by an incident plane wave,

$$\phi^{(i)} = \frac{gA}{\sigma} e^{-ky} \cos(kx - \sigma t), \qquad k = \sigma^2/g$$

incident on a rigid cylinder. In this instance we take

$$\left(\frac{\partial^2 \phi}{\partial \rho \partial t}\right)_{\rho=a} = gAk\, e^{-ka\sin\theta} \cos(ka\cos\theta + \theta - \sigma t);$$

the term in the force multiplying $\cos\sigma t$ is odd with respect to $\theta = \pi/2$, and

$$F_v^{(2)} = -2k\mu gAa^2 \int_0^{\pi/2} D(\theta)\sin\theta\, d\theta \,\sin\sigma t, \tag{16}$$

where $D(\theta) = e^{-ka\sin\theta} \sin(ka\cos\theta + \theta)$

then represents the force due to the diffracted wave. To solve the problem of the motion of the cylinder due to an incident wave, we combine these last two results. We assume the motion of the cylinder is

$$\zeta = \zeta_0 \sin(\sigma t + \epsilon).$$

For the total velocity potential we take

$$\phi = \phi^{(1)} + \phi^{(2)} + \phi^{(i)},$$

where $\phi^{(1)}$ is the potential due to motion of cylinder, and $\phi^{(2)}$ is the potential of the diffracted wave, from a rigid cylinder. By construction we then have

$$\frac{\partial \phi^{(2)}}{\partial r} = \frac{\partial \phi^{(i)}}{\partial r} \qquad \text{at} \qquad r = a;$$

and, hence,

$$\frac{\partial \phi}{\partial r} = \frac{\partial \phi^{(1)}}{\partial r} = \zeta_0 \sigma \cos(\sigma t + \epsilon)\cos\theta \qquad \text{at} \qquad r = a,$$

which is the desired boundary condition. On combining Equation (3) of the previous section with Equation (15) and (16) we can compute the vertical force on the cylinder. The equation of motion then becomes

$$\mu \pi a^2 \sigma^2 \zeta_0 \sin(\sigma t + \epsilon) = -\left\{ 2\mu g k A a^2 \int_0^{\pi/2} D(\theta) \sin\theta \, d\theta + 2\mu g A a f(ka) \right\} \sin \sigma t$$

from which it follows that $\epsilon = 0$, and

$$|\zeta_0 / A| = \frac{2}{\pi} \left\{ \int_0^{\pi/2} D(\theta) \sin\theta \, d\theta + \frac{f(ka)}{ka} \right\} . \tag{17}$$

The conclusion $\epsilon = 0$ means that the motion of the cylinder is either in phase, or exactly 180° out of phase with the pressure due to the incident wave at $x = 0$.

The results from Equation (17) are plotted on Figure 2 for the region of higher frequencies. For smaller frequencies, the results cease to be of value but it is seen that between the two approximate theories included in this report, it is possible to cover almost the entire range of frequencies with considerable accuracy.

For purposes of comparison the results using values of the virtual mass and damping factors which are constant and equal to their experimentally determined values are included in Figure 2. It is seen that the maximum amplitude predicted in this manner is nearly 20 percent too high, while the method of this report yields the maximum almost exactly.

5. Future Studies

The analysis of the present report may be repeated for the case of the sphere, although the procedures are somewhat more complicated. The solution for large σ has been obtained and the numerical calculations have been carried out. The formal solution to the forced oscillation problem has also been obtained and calculations are under way. The calculations involve solutions of systems of linear equations and are quite lengthy.

It may be remarked that despite its apparent complications, the problem of the sphere seems to be fundamentally simpler than that of the cylinder. The reason is that the wave motion due to the presence of the sphere vanishes at infinity(3) and one is left with the incident wave only. The cylinder on the other hand produces waves which remain finite at infinity and it is not clear what one should take as conditions at infinity.

It is in fact possible to obtain formally a complete solution to the problem of the motion of a sphere in an incident wave. This solution has been obtained and involves some additional numerical calculations, in addition to which there are serious convergence questions still unanswered. It is however, planned to present these results in the near future.

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Appendix - Discussion of the Boundary Condition

In order to study the simplified boundary condition, $\phi = 0$, we first consider the exact solution of the problem of waves of frequency, σ, in the presence of obstacles of bounded cross section. Suppose that the depth of the water is finite, h, and let R denote the region of the fluid exterior to the obstacle. In this region the potential, ϕ, may be expanded in a series of characteristic functions,

$$\phi = A_0 \cos k_0(x-x_0)\cosh k_0(y+h) + \sum_1^n A_n \cos k_n(y+h)\cosh k_n(x-x_0), \tag{A-1}$$

where k_0, $\pm i k n$ are roots of the equation,

$$\sigma^2 = g\,k \tanh k\,h$$

and the A_n are constants. These constants could be determined if ϕ were known over the vertical section $x = x_0$, $-h < y < 0$. However, for our purpose it suffices to remark that, in water of large depth, the potential quickly approaches the form

$$\phi = A_0 \cos k_0(x-x_0)\cosh k(y+h)$$

as the distance from the obstacle increases. Hence, we have

$$\phi = O(A) \quad \text{for} \quad y = 0\,. \tag{A-2}$$

Now the surface elevation, η, is obtained from ϕ by means of the equation

$$\eta = \frac{1}{g}\frac{\partial\phi}{\partial t}$$

which, in the case of periodic motion, means

$$\eta \doteq O\left(\frac{\sigma}{g} A_0\right)\,. \tag{A-3}$$

Now let a be a dimension of the obstacle. We may determine the dependence of A_0 and consequently of ϕ and η on σ by a simple dimensional argument. The variables of the problem may be taken as a, σ and g. We then set

$$A_0 = O(g^{\alpha}\ \sigma^{\beta}\ a^{\gamma})$$

and determine α, β, γ so that A_0 has the proper dimensions, namely $\frac{(\text{length})^2}{\text{time}}$. This calculation gives

$$A_0 = O(g^{\alpha}\ \sigma^{1-2\alpha}\ a^{2-\alpha})$$

for some α. Thus we have

$$\phi = O(g^{\alpha}\ \sigma^{1-2\alpha}\ a^{2-\alpha}) \quad \text{for} \quad y = 0,\ \eta = O(g^{\alpha-1}\ \sigma^{2-2\alpha}\ a^{2-\alpha})\,. \tag{A-4}$$

Now the velocity potential of a progressive wave is given by

$$\phi = \frac{gA}{\sigma} \frac{\cosh k_0 (y + h)}{\cosh k_0 h} \cos(k_0 x - \sigma t), \tag{A-5}$$

where A is an amplitude. For the cases in which we are interested, namely, the motion produced by forced oscillation of a cylinder, and by the diffraction of an incident wave about such a cylinder, we wish to have the wave motion at infinity be of the form in Equation (A-5), with A bounded. It follows then, by comparing Equations (A-4) and (A-5), that

$\alpha = 1$; that is,

$\phi = O(1/\sigma)$ at $y = 0$, $\eta = O(1)$.

We have shown, therefore, that to obtain motions produced by an obstacle of bounded cross-section, which give finite wave heights at infinity, we should solve the problem obtained by setting $\phi = 0$ on the free surface.

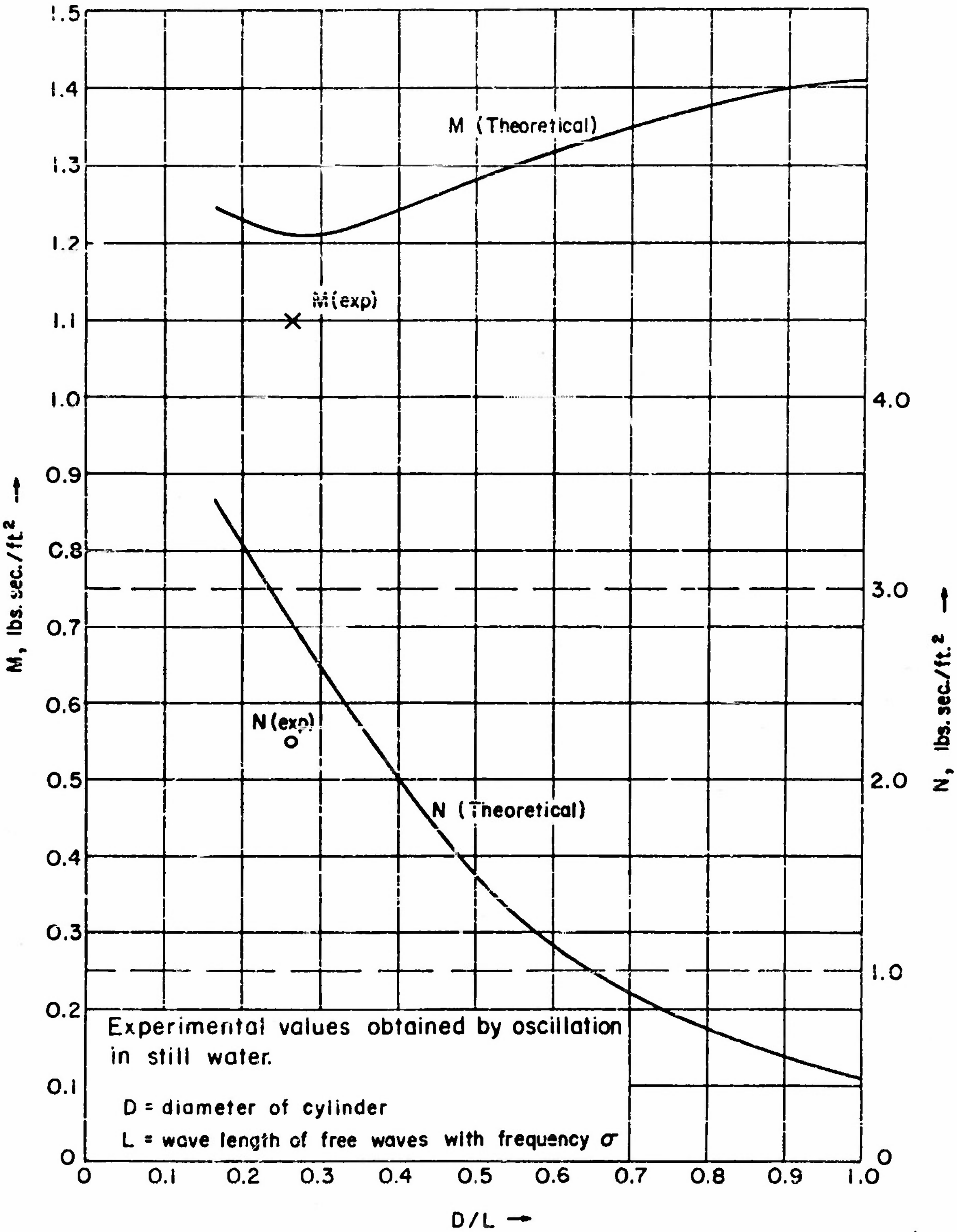

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FIGURE 1

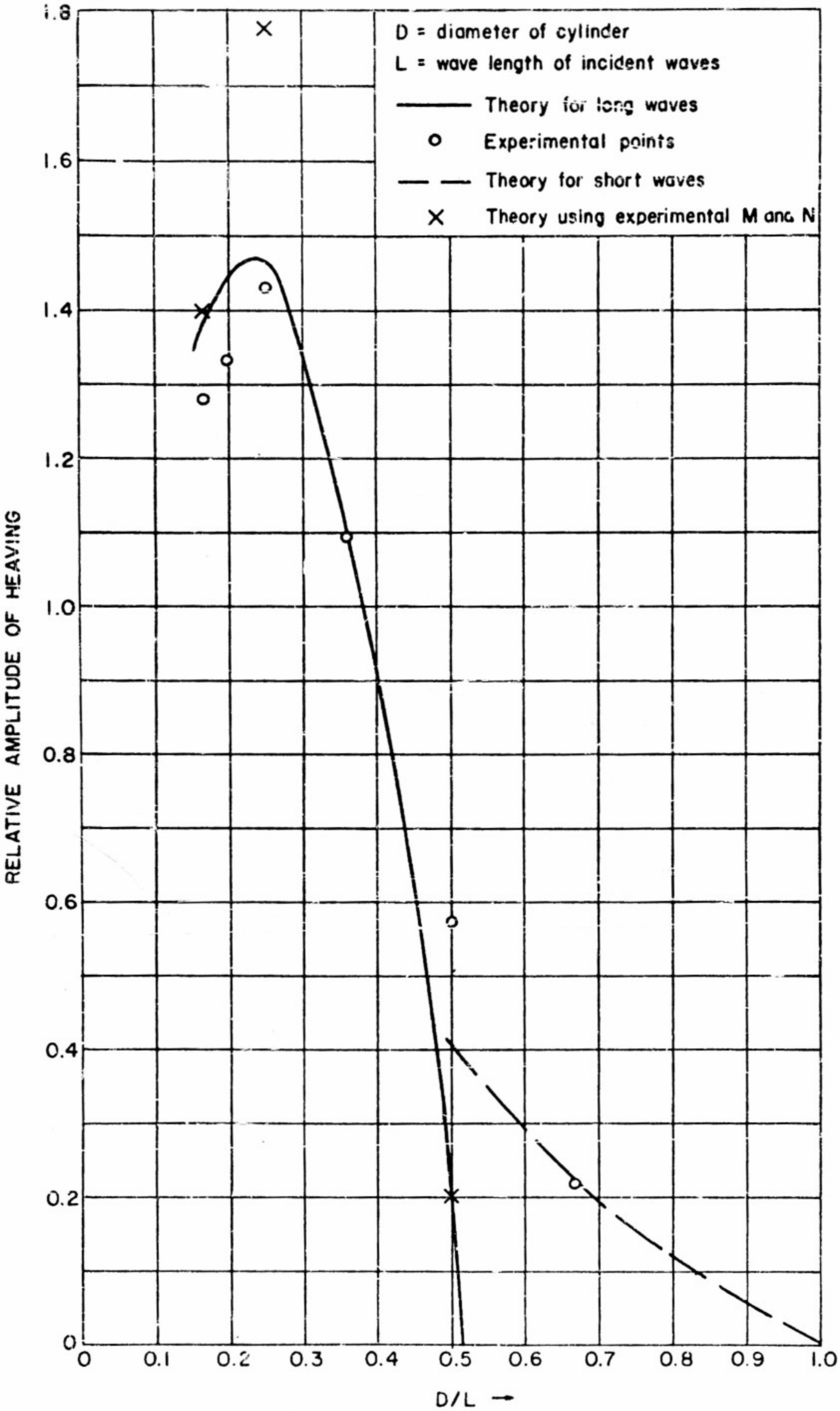

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FIGURE 2

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